Interleaving Radiosity

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Abstract A system of linear equations is in general solved to approximate discretely the illumination function in radiosity computation. To improve the radiosity solution, a method that performs shooting and gathering in an interleaving manner is proposed in the paper. Besides, a criterion has been set up and tested for choosing object elements used in the gathering operation, and a criterion is established to quantify the solution errors by taking into account more reasonably of the human perception of the radiosity solution. Experimental results show that the method proposed has nice performance in improving the radiosity solution.

Keywords radiosity, global illumination, progressive refinement, shooting and gathering, interleaving

1 Introduction

The calculation of global illumination in computer graphics is based on the theory of radiance transport. The general equation of radiance transport[1,2] is:

$$\frac{dI(X,\omega)}{dx} = -\kappa_a(X)I(X,\omega) + \kappa_t(X)I_e(X) + \frac{\kappa_s(X)}{4\pi}\int_S r(X,\omega',\omega)I(X,\omega')d\omega' \tag{1}$$

where

1) X is a position in three-dimensional space; ω and ω' are ray directions on the unit sphere;

2) $I(X,\omega)$ is the radiance at position X flowing along a ray in direction ω and $I_e(X)$ is the source at position X, representing the local production of radiance;

3) $r(X,\omega',\omega)$ is the scattering phase function from direction ω' to direction ω, the integral is over the 4π solid angle of incoming direction ω' at X;

4) $\kappa_t(X) = \kappa_a(X) + \kappa_s(X)$ is the extinction or beam attenuation coefficient representing absorption (κ_a) plus scattering (κ_s).

In particular, the rendering equation could be established by applying the radiant transport equation to the illumination of natural environment[3]. Based on the principle of heat transfer, Goral *et al.* introduced the radiosity method as a way to approximately solve the global illumination problem for image synthesis[4]. The solution is to apply the transport equation to a diffuse geometrical environment composed of plane surfaces where the integral kernel $r(X,\omega',\omega)$ in (1) depends only on the position (X) and direction of the surfaces, and the absorption of radiance is neglected.

To obtain a radiosity solution, one has to solve the following system of linear equations that results from a discrete approximation of the illumination function[5],

$$B_i = E_i + \rho_i \sum_j B_j F_{ij},\ i = 1,\ldots,n \tag{2}$$

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where B_i is the radiosity of the ith element, ρ_i is the reflectivity of the ith element, E_i is the emmision of the ith element, and F_{ij} is the form factor from element i to element j (the fraction of power leaving element i that arrives directly at element j).

The linear set of (2) could be expressed in the matrix form as

$$\boldsymbol{MB} = \boldsymbol{E}$$

where $\boldsymbol{B}, \boldsymbol{E}$ are column vectors of B_i, E_i, respectively, and $\boldsymbol{M}$ is an $n \times n$ matrix with $M_{ij} = \delta_{ij} - \rho_i F_{ij}$, here δ_{ij} is the Kronecker symbol, whose value is 1 if and only if $i = j$.

2 Previous Solutions

Since the matrix $\boldsymbol{M}$ in the linear system is diagonally dominant[7−9], the following *Gauss-Seidel* (GS) iteration has been suggested to solve the linear system of equations:

1. for all i
2. $\quad B_i = E_i$
3. endfor (i)
4. while not converged
5. $\quad$ for each i in turn
6. $\quad\quad B_i = E_i + \rho_i \sum_{j \neq i} B_j F_{ij}$
7. $\quad$ endfor(i)
8. endwhile
9. display the image using B_i as the intensity of element i.

The physical interpretation of the algorithm is that, along with GS iteration each element (i) gathers the radiosity from other elements (at line 6), using the most recent estimates (B_j).

Cohen *et al.* introduced the progressive refinement radiosity (PR) algorithm as an alternative to solve the linear system[6]. PR has the advantages of quickly converging to an accurate image and producing an image at early stage that is good enough for preview, while the computation proceeds[11]. The procedure of the PR algorithm is as follows:

1. for all i
2. $\quad B_i = E_i$
3. $\quad \Delta B_i = E_i$
4. endfor (i)
5. while not converged
6. $\quad$ pick i, such that $\Delta B_i * A_i$ is the largest
7. $\quad$ for every element j
8. $\quad\quad \Delta R_j = \Delta B_i * \rho_j (A_i/A_j) F_{ij}$
9. $\quad\quad \Delta B_j += \Delta R_j$
10. $\quad\quad B_j += \Delta R_j$
11. $\quad$ endfor (j)
12. $\quad\quad \Delta B_i = 0.0$
13. endwhile
14. display the image using B_i as the intensity of element i.

In this algorithm, during one iteration (lines 6 ~ 13), the element i with the largest unshot radiosity shoots its radiosity onto all the other elements in the environment. Thus, $O(n)$ operations are required in one shooting step to update all the other elements, where n is the number of elements in the environment.

Overshooting was proposed to accelerate the convergence in the progressive refinement method[11]. By overshooting, an estimate of future reflected radiosity, in general an ambient term is pre-added to the current unshot radiosity for shooting. Feda *et al.*[11] defined the overshooting part to be the area-weighted average unshot radiosity increased by the geometric series of the area-weighted average reflectivity, taking into account of multi-reflections. Gortler *et al.*[9] used the predicted radiosity to be regathered as the term of an element from the radiosity of all the other elements incurred by the shooting of current unshot radiosity of the element. In the original progressive refinement approach proposed by Cohen *et al.*[6], the estimated ambient term of the environment was used for display purpose but not as a part of the iterative solution.

3 Interleaving of Shooting and Gathering

The gathering method is actually Gauss-Seidel iteration to the linear system and the progressive refinement shooting method is the Southwell relaxation plus a final Jacobi sweep[6,9,10]. Mathematical analysis and practical experience show that[6,9] PR method converges more quickly, particularly at the early stage of iteration than GS solution, because Southwell relaxation has the advantage of choosing the largest residual, expecting to reduce the total residual by a larger amount. This property suits well when the initial residual is concentrated at the light sources, i.e., only few nonzero terms exist in the emission vector $\boldsymbol{E}$. Therefore at the early stage of iteration Southwell's relaxation method can concentrate its effort on the variables that change by the largest but Gauss-Seidel spends much time updating variables that do not change very much. However, on the other hand, as Gortler *et al.* pointed out[9], "this fact does not imply that Southwell must always do better than GS in the long run, and in some cases, making a 'worse' choice now would let us remove more error in later steps". The statistics of various examples[9] also showed that, while the number of the iteration steps is getting larger the performance of PR method in comparison with GS method is getting closer, despite the fact that in most cases PR method apparently outperforms Gathering method due to the fact that the shooting elements in PR are in general only a quite small part of the total elements.

Measuring the error of a solution is critical in radiosity computation. On this aspect, the development of perceptual error metrics for image synthesis has to be emphasized. The errors of a solution of the linear system could be quantified by its difference to the exact solution in terms of norms used, such as L_0, L_1 or L_2[12,13]. However, fast convergence in a quantitative error metric does not necessarily imply fast convergence of the resulting images, as perceived by a human observer[12]. Although the precise quantitative feature of human perception is still an open problem for the future study, at least we know that, the function of the perceived linear brightness is logarithmic with respect to actual intensity, similar to the monitor response curve[14]. Thus from

$$C = a\log(I)$$

where C and I are perceived color and light intensity, respectively, and a is a constant, we have

$$dC = a\cdot(dI/I) \text{ or } \Delta C \sim (\Delta I/I)$$

In other words, it is the relative error of intensity or radiance that should be more precisely used to express the error perceived by a human observer. Therefore in the following discussion as well as in our test examples we will use this definition of error for the error quantification. We will call this error as perceived error, or simply p-error.

Let us consider again the PR approach and the linear equation

$$B_k = E_k + \rho_k \sum_{j=1}^{n} B_j(A_j/A_k)F_{jk} \tag{3}$$

for an arbitrary non-light source element k in the radiosity system (2). Suppose that altogether m elements have been used for shooting, i.e., m items among total n on the right side of (3) have been calculated, with each non-zero item contributed from each shooting. Without loss of generality, we assume that these elements are indexed from 1 to m. Then the radiosity of element k from the PR solution is

$$B'_k = E_k + \rho_k \sum_{j=1}^{m} B'_j(A_j/A_k)F_{jk}$$

Therefore, the p-error of element k could be roughly estimated by

$$p_k = \frac{\rho_k \sum_{j=m+1}^{n} B_j(A_j/A_k)F_{jk}}{\rho_k \sum_{j=1}^{m} B_j(A_j/A_k)F_{jk} + \rho_k \sum_{j=m+1}^{n} B_j(A_j/A_k)F_{jk}} \approx \frac{\sum_{j=m+1}^{n} B_j(A_j/A_k)F_{jk}}{B'_k/\rho_k + \sum_{j=m+1}^{n} B_j(A_j/A_k)F_{jk}}$$
$$= \frac{D_k}{B'_k/\rho_k + D_k}$$

where $D_k = \sum_{j=m+1}^{n} B_j(A_j/A_k)F_{jk}$. This is an increasing function of D_k, and $\rho_k D_k$ is roughly the radiosity that has not been shot yet from the remaining unshot elements. If we assume that the subdivision of objects is uniform, or the size of elements has no large variation, then we have

$$D_k \approx \sum_{j=m+1}^{n} B_j F_{jk}$$

Statistically, the sum of B_j for j from $m+1$ to n on average is much smaller than that for j from 1 to m, as the later is summed from the shooting elements, including light sources. However, in most cases, the number of unshooting elements $(n-m)$ is much larger than that of shooting elements (m). Therefore, $\Omega_k = \sum_{j=m+1}^{n} F_{jk}$ would be an important factor in determining the value of D_k, accordingly the p-error. Geometrically, the value of Ω_k is directly related to the solid angle at element k, which has not yet been covered by the shooting elements. This implies that, after a shooting session with a given number of shootings, the elements whose solid angles covered by shooting elements are small tend to produce large errors in the solution.

Previous analysis suggests an improvement to the radiosity solution that, after a series of shootings, if a gathering operation is performed to those elements whose receipt of radiosity was not adequate during the shooting session, the solution error (p-error) could be reasonably reduced. Based on this observation, a method is proposed and implemented by conducting shooting and gathering in an interleaving manner: each shooting session is followed by a gathering session in the solution process. The procedure can be expressed in pseudo-code by:

for all i
  $B_i = E_i$
  $\Delta B_i = E_i$
endfor(i)
while not converged
 starting a shooting session by first sorting elements according to ΔB_i
 for all i, $\Delta ff_i = 0.0$, endfor(i)
 pick i, such that $\Delta B_i * A_i$ is the largest
   for every element j
     $F_{ji} = (A_i/A_j)F_{ij}, \Delta ff_j + * = F_{ji}$
     $\Delta R_j = \Delta B_i * \rho_j F_{ji}$
     $\Delta B_j + = \Delta R_j$
     $B_j + = \Delta R_j$
   endfor(j)
   $\Delta B_i = 0.0$, $\Delta ff_i = C$ (a large constant)
  Reserve the unshooting radiosity
  starting a gathering session by sorting elements according to Δff_j
  pick r, such that Δff_r is the smallest
    – element r is selected for gathering radiosity from other elements
   for every element j that did not conduct shooting in the shooting session
     $\Delta R_r = \Delta B_j * \rho_r F_{rj}$
     $\Delta B_r + = \Delta R_r$
     $B_r + = \Delta R_r$
     $\Delta ff_r = C$ (a large constant)
   endfor(j)

25. endwhile
26. display the image using B_i as the intensity of element i.

Here for each element i, a variable Δff_i is defined for accumulating the form-factors or solid angles of element i covered by the shooting elements. In order to prepare for the radiosity gathering in the gathering session, for each shooting in the shooting session, the solid angle of the shooting element i with regard to the receiving element j will be accumulated into the variable Δff_j of element j (see line 10), to be used at the start of the gathering session for element sorting (see line 17). The sorting operation is conducted according to the increasing order of Δff_j, and as a result, the element with the least value of Δff would be the first to gather radiosity in the gathering session.

Clearly, calculation of n form-factors is required for completing radiosity gathering for an element. This shows that one step of gathering takes roughly equal computation to that of shooting one element as both require the same number of form-factor computations. This has been confirmed by the experimental statistics of test examples.

In the implementation of interleaving radiosity method, in order to preserve light energy among the objects within the environment of radiosity solution, two additional messages have to be added to the original data structure of the PR shooting method for each patch. One is an integer called *Gathering No.* standing for the number of the most recent session during which the related patch undertook energy gathering. For most of the patches this number should be zero because only a small number of patches participate in the gathering process. Another message is to extend the original ΔB_i for each patch i to a small array of $\Delta B_i[\,]$, with each element having a record of the accumulated energy received at each shooting session. Thus for each shooting at each shooting session, a proper shooting energy can be received by the receiving patch according to the *Gathering No.* of the patch. However, in most cases, the shooting energy at the present layer should be received by the shot patches as usual like in the normal PR method so that the penalty in the interleaving approach in the implementation would be minor.

4 Result and Analysis

Two examples were tested to show the result of the method proposed in the paper. The first example (example A) is a closed space of a computer room, composed of 4,953 elements for radiosity calculation, and the second one (example B) is a simple environment of a room, with 1,336 elements divided for calculation. In order to evaluate the solution errors, the exact solution was firstly obtained through an iterative method of calculation in an extremely high precision to meet the set of equations. Then the difference of various solutions with the exact solution in terms of *p-error* was calculated.

Theoretically, as the matrix of the linear system is diagonally dominant and all the elements in the matrix are non-negative, the intermediate solution of the PR method should converge to the exact solution as the shooting process goes on. However, due to the addition of the approximate value of the ambient term or predicted radiosity term applied in various solutions, as well as the various numerical errors arising during the shooting process, the solution cannot be further improved after certain stages. Situation could become even worse that over-shooting may be cumulated on some part of the environment. The dashed lines in Fig.1 show the *p-errors* of the intermediate shooting solution for the example A, starting from the number of shootings 420 ((a), (b) and (c) are, respectively, for red, green and blue colors which stands in other figures). The shooting numbers for coming up with the bad behavior (we call them turning numbers afterwards) in this example are roughly 800, 700, 700, respectively, for red, green and blue colors. Similar behavior is demonstrated in the example B as shown in Fig.3. In this example, the turning numbers are 1000, 650, 600 respectively for red, green and blue colors. Fig.5 and Fig.7 show the *p-error* in the examples A and B respectively by a display of *p-error* distribution in colors. Here the green color indicates negative *p-error*, or under-illumination, and the yellow color shows positive *p-error*, or over-illumination. The three columns of pictures in Fig.5 (Exp. A) show the *p-errors* for the result of radiosity solution with the shooting numbers 400, 600 and 800 respectively, and in Fig.7 (Exp. B) 1,020, 1,220, and 1,400, respectively.

We may see from the figures that, some part in the environment obviously demonstrates more and more over-illuminated effect along with the shooting process, even long before the shooting procedure reaching to turning numbers, say in the example A, both 700 for green and blue colors, though most part shows nice behavior in gradually approaching to the exact solution.

The rationality of the interleaving approach could be justified from Fig.2 and Fig.4, respectively for examples A and B after the number of shootings in 400, 600 and 800 for example A and 600, 800 and 1,000 for example B, respectively. In these two figures, the functional relationship between the sum of form-factors (*ffas*) and *p*-errors of individual elements is illustrated. The horizontal axis (*ffa*) in the coordinates represents the sum of form-factors of geometric elements which have been shot by the shooting elements after the shooting process. In order to generate the functional relation statistics, for all the elements in the shooting process we first accumulate the form-factors that were shot by the shooting element in each shooting. Then we sort the sum of form-factors (*ffas*) in an increasing order

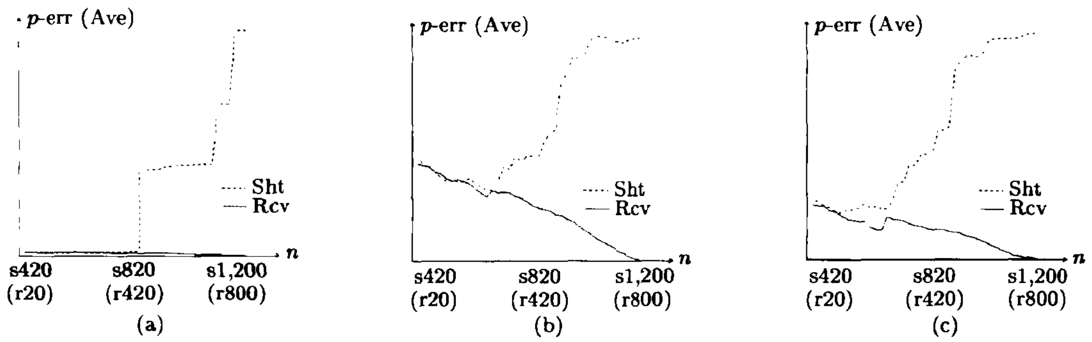

Fig.1. Comparison of *p*-errors for Exp. A between pure shooting (dashed) and interleaving (solid).

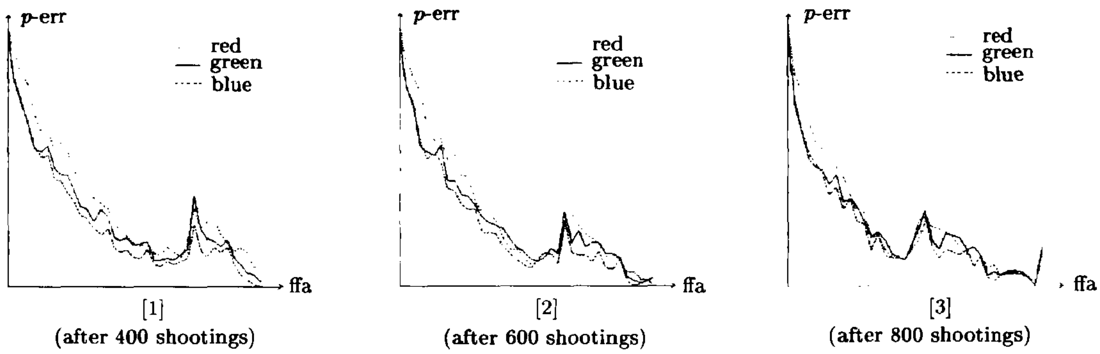

Fig.2. Distribution of *p*-error against *ffa* among the environment objects for Exp. A.

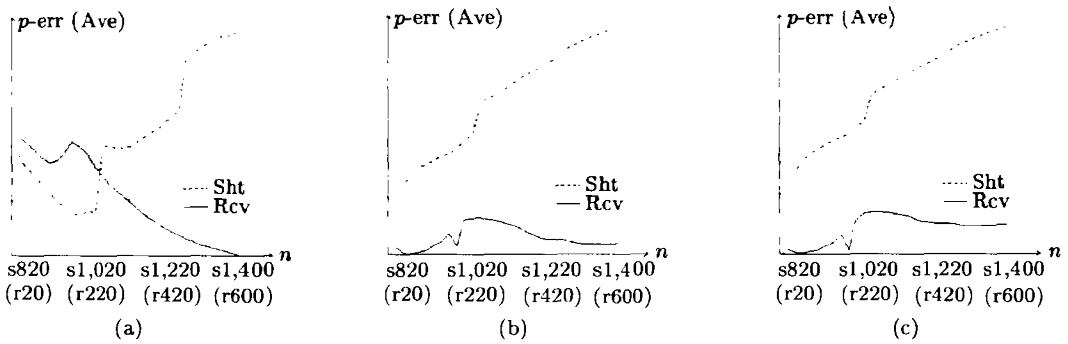

Fig.3. Comparison of *p*-errors for Exp. B between pure shooting (dashed) and interleaving (solid).

for all the elements. The p-errors of the elements in correspondence with ffas are shown in the vertical coordinates in Fig.2 and Fig.4. The result convincingly shows that bigger p-errors appear on the elements that possess less *ffas*. This is the reason that in the receiving process, good performance can be achieved by choosing the elements with the least *ffa* in an environment. The geometric elements among the starting part are substantial to the solution convergence in the energy receiving stage. The solid lines in Fig.1 and Fig.3 show the improved performance in terms of p-errors through a radiosity receiving process after a shooting process, in comparison with the corresponding dashed lines showing the performance of a single continuing shooting process.

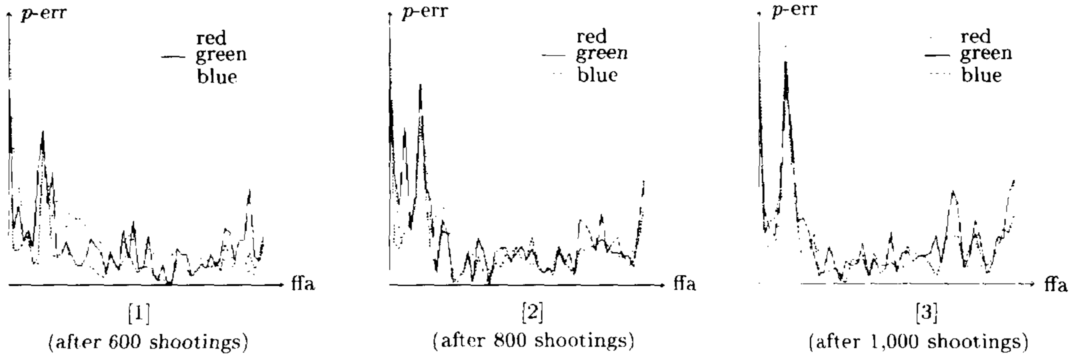

Fig.4. Distribution of p-error against *ffa* among the environment objects for Exp. B.

Fig.7 and Fig.8 give a comparison of *p-error* distribution between pure shooting and the interleaving, respectively for examples A and B. In example A (Fig.7) comparison of 800 shootings (top) with 400 shootings and 400 receivings (bottom) is given, and in example B (Fig.8) comparison of 1,020 shootings (1st column) with 800 shootings and 220 receivings (2nd column), 1,400 shootings (3rd column) with 800 shootings and 600 receivings (4th column) is given. The figures show that, though on some parts (e.g., the ceiling in Exp. A) the pure shooting has better result, but as a whole the solution error arising from the interleaving method is uniformly distributed on the whole environment. On some of the over-illuminated parts the interleaving method improves the solution much better than the pure shooting method does, in particular, say for green and blue color components in the example A.

Fig.9 and Fig.10 show the environment display from the radiosity solution. Four pictures in Fig.9 show the result of example A for precise solution (top left), pure shooting of 200 (top right), 800 (bottom left), and 400 shootings with 400 receipts (bottom right) respectively. Four pictures in Fig.10 show the result of example B for exact solution (top left), pure shooting of 200 (top right), 1,400 (bottom left), and 800 shootings with 600 receipts (bottom right), respectively. (Please refer to the inserted leaf at the end of this paper for Figs.5 – 10.)

5 Conclusion

In order to improve the radiosity solution, we propose and implement a method in which the shooting process and gathering process are interleaved, or a series of cycles with each cycle in combination of a shooting process and a gathering process are performed for the radiosity solution. In particular, a criterion has been set up and approved for choosing the object elements participating in the gathering operation. This is based on sorting the sum of form-factors that had been shot in total in the previous shooting process.

The primary idea of the interleaving radiosity is to improve the overall performance of the progressive refinement radiosity in the pure shooting method, through quickly collecting energy by a following gathering process for part of environment elements that lag behind in the shooting process. However, in general, the shooting process among the shooting and gathering processing is still dominant in the interleaving radiosity. There must be a balance point in terms of the ratio of the number of shooting patches to that of gathering patches in each session in this regard. The balance point should certainly depend on the environment parameters, in particular the object and light source distribution of the environment. The problem is expected to be an open issue for the further study.

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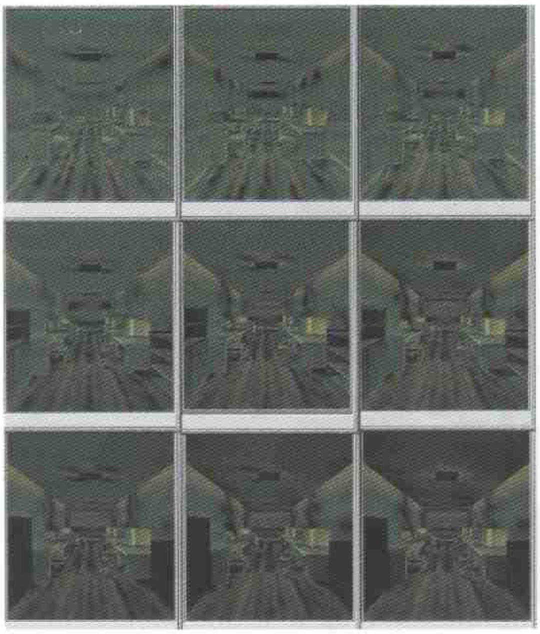
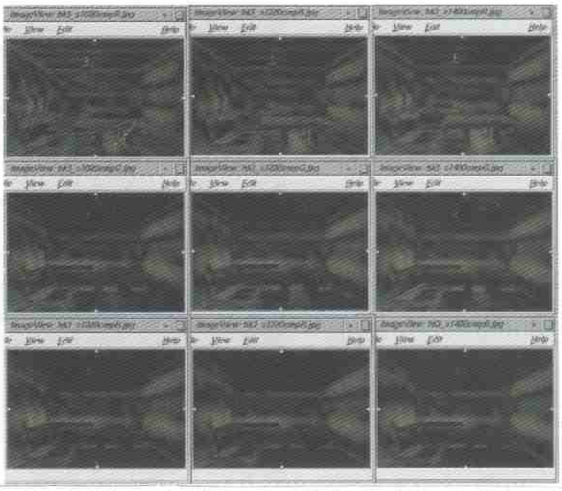


(after 400 shts) (after 600 shts) (after 800 shts)

(Top: for Red; Mid.: for Green; Bottom: for Blue)

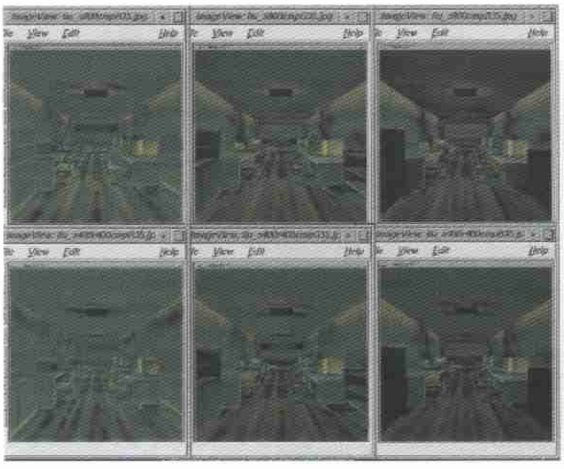
Fig.5. p-error distribution (in colors) for Exp. A.



(after 1,020 shts) (after 1,220 shts) (after 1,400 shts)

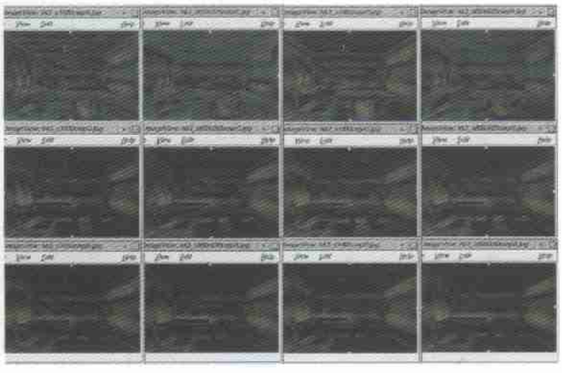
(Top: for Red; Mid.: for Green; Bottom: for Blue)

Fig.6. p-error (in colors) distribution for Exp. B.



(Red) (Green) (Blue)

Fig.7. Comparison of p-error distribution between pure shooting and interleaving (Exp. A).



(Top: for Red; Mid.: for Green; Bottom: for Blue)

Fig.8. Comparison of p-error distribution between pure shooting and interleaving (Exp. B).

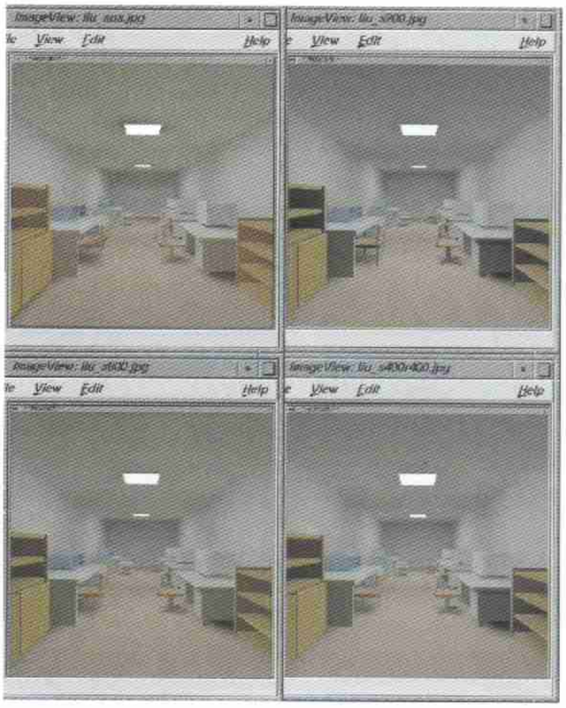


Fig.9. Environment display of radiosity solutions (Exp. A).

Fig.10. Environment display of radiosity solutions (Exp. B).